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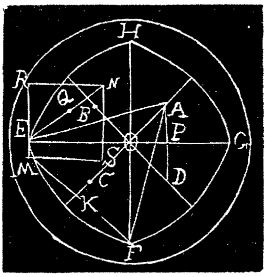


to itself; Q will describe the figure $EFGH$, whose boundary consists of the four equal arcs, EF, FG, GH, HE, whose centers are A, B, C, D, and radius equal to that of the given circle.

Now while MN is given in length and direction, the area of the figure $EFGH$ represents the number of ways the two points can be taken, so that the square will be wholly within the circle.

Let $AE=r$, $MN=2x$, $\angle AEP=\theta$, $\phi=$the angle which MN makes with some fixed line, area $EFGH=u$.

Then we have $OA=x$, $AP=\frac{1}{2}x\surd 2$, $\angle EAK=\angle AOP-\angle AEP=\frac{1}{4}\pi-\theta$, $\frac{1}{2}x\surd 2=r\sin\theta$, $EK=r\sin(\frac{1}{4}\pi-\theta)$, area $EOF=r^2(\frac{1}{4}\pi-\theta)-rx\sin(\frac{1}{4}\pi-\theta)=r^2(\frac{1}{4}\pi-\theta-\sin\theta\cos\theta+\sin^2\theta)$, and $u=r^2(\pi-4\theta-4\sin\theta\cos\theta+4\sin^2\theta)$. An element of the circle at the point N is $4xdxd\phi$, or $8r^2\sin\theta\cos\theta d\theta d\phi$. The limits of θ are 0 and $\frac{1}{4}\pi$; and of ϕ, 0 and 2π. Hence the chance in question is

$$\delta=\frac{1}{\pi^2r^4}\int_0^{\frac{1}{4}\pi}\int_0^{2\pi}r^2(\pi-4\theta-4\sin\theta\cos\theta+\sin^2\theta).8r^2\sin\theta\text{cas}\theta d\theta d\phi$$

$$=\frac{16}{\pi}\int_0^{\frac{1}{4}\pi}(\pi-4\theta-4\sin\theta\cos\theta+4\sin^2\theta)\sin\theta\cos\theta d\theta=2-(4/\pi).$$

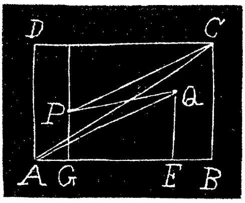
An excellent solution of this problem was also received from *G. B. M. ZERR*.

136. Proposed by L. C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.

By direct computation find the average distance between two points in the surface of a given rectangle, but on the opposite sides of a diagonal.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa., and the PROPOSER.

Let P, Q be the random points, $AB=a$, $BC=b$, $a^2+b^2=d^2$, $AE=x$, $QE=y$, $CF=u$, $PF=v$. Then $PQ=\surd[(x+u-a)^2+(y+v-b)^2]$. The limits of x are 0 and a; of u, 0 and a; of y, 0 and bx/a; of v, 0 and bu/a. Let $(x+u-a)=c$, $\triangle=$average distance.



$$\therefore\triangle=\frac{4}{a^2b^2}\int_0^a\int_0^a\int_0^{bx/a}\int_0^{bu/a}\surd[c^2+(y+v-b)^2]$$

$$\times dxdudydv=\frac{2}{a^2b^2}\int_0^a\int_0^a\int_0^{bx/a}\Big[\Big(\frac{ay+bu-ab}{a^2}\Big)$$

$$\times\surd(a^2c^2+(ay+bu-ab)^2]-(y-b)\surd[c^2+(y-b)^2]$$

$$+c^2\log\Big(\frac{ay+bu-ab+\surd[a^2c^2+(ay+bu-ab)^2)}{ay-ab+a\surd[c^2+(y-b)^2]}\Big)\Big]dxdudy$$

$$=\frac{2}{3a^5b^2}\int_0^a\int_0^a\{[2a^2c^2-b^2(u-a)^2]\surd[a^2c^2+b^2(u-a)^2]+[(2a^2c^2-b^2)(x-a)^2]$$

$$\times\sqrt{[a^2c^2+b^2(x-a)^2]}-a^3(2c^2-b^2)\sqrt{[b^2+c^2]}+dc^3(d^2-3a^2)$$

$$+3a^2bc^3\log[c(b+d)]-3a^3bc^2\log a-3a^2bc^2(x-a)\log[b(x-a)$$

$$+\sqrt{[a^2c^2+b^2(x-a)^2]}]-3a^3bc^2\log[\sqrt{(b^2+c^2)}-b]-3a^2bc^2(u-a)\log[b(u-a)$$

$$+\sqrt{[a^2c^2+b^2(u-a)^2]}]\}dxdu$$

$$=\frac{1}{72a^5b^2}\int_0^a\Big[24x[2a^2x^2-3b^3(x-a)^2]\sqrt{[a^2x^2+b^2(x-a)^2]}-16a^3bx^3$$

$$-16a^3b(x-a)^3+(x-a)^4(72b^2d-12a^2d-12d^3-23a^2b)+x^4(12d^3-36a^2d$$

$$+23a^2b)+12a^3x(3b^3-2x^2)\sqrt{[b^2+x^2]}+12a^3(x-a)[2(x-a)^2-3b^2]$$

$$\times\sqrt{[b^2+(x-a)^2]}+60a^2b(x-a)^4\log[(b+d)(x-a)]$$

$$+\frac{24b^4(x-a)^4}{a}\log[(a+d)(x-a)]-96a^2bx^3(x-a)\log\{b(x-a)$$

$$+\sqrt{[a^2x^2+b^2(x-a)^2]}\}-48a^3bx^3\log[\sqrt{(b^2+c^2)}-b]-\frac{24b^4(x-a)^4}{a}$$

$$\times\log\{ax+\sqrt{[a^2x^2+b^2(x-a)^2]}\}+48a^3b(x-a)^3\log\{\sqrt{[b^2+(x-a)^2]}-b\}$$

$$+12a^3b^4\log[x+\sqrt{(b^2+x^2)}]-12a^3b^4\log\{x-a+\sqrt{[b^2+(x-a)^2]}\}$$

$$+36a^2bx^4\log[x(b+d)]-48a^3bx^3\log a+48a^3b(x-a)^3\log a\Big]dx$$

$$=\tfrac{1}{15}\Big[2d\Big(2-\frac{a^2}{b^2}-\frac{b^2}{a^2}\Big)-\frac{a^3+b^3}{d^2}+\frac{2a^3}{b^2}+\frac{2b^3}{a^2}+\Big(\frac{4b^2}{a}-\frac{a^2b^2}{d^3}\Big)\log\Big(\frac{a+d}{b}\Big)$$

$$+\Big(\frac{4a^2}{b}-\frac{a^2b^2}{d^3}\Big)\log\Big(\frac{b+d}{a}\Big)\Big]=\frac{a}{30}[6+(16-\sqrt{2})\log(1+\sqrt{2})],\text{ if } a=b.$$

MISCELLANEOUS.

135. Proposed by LON C. WALKER, A. M., Graduate Student, Leland Stanford University, Cal.

Find invariants of the second, third, and sixth degrees in the coefficients of a binary quartic.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $ax^4+4bx^3y+6cx^2y^2+4dxy^3+ey^4=u=0$, be the binary quartic.

Then $d^4u/dx^4=24a$, $d^4u/dx^3dy=24b$, $d^4u/dx^2dy^2=24c$, $d^4u/dxdy^3=24d$, $d^4u/dy^4=24e$.